13. Gravitational Fields

13.2 Gravitational force between point masses

Paper 4

Question Paper

1 **(a)** State Newton's law of gravitation.

.....

.....

..... [2]

(b) A planet may be considered as a uniform sphere.

A satellite is in circular orbit of period T around the planet at a height h above the surface. The height of the orbit can be adjusted by use of the satellite's rocket engines.

Fig. 1.1 shows the variation with h of $T^{\frac{2}{3}}$.

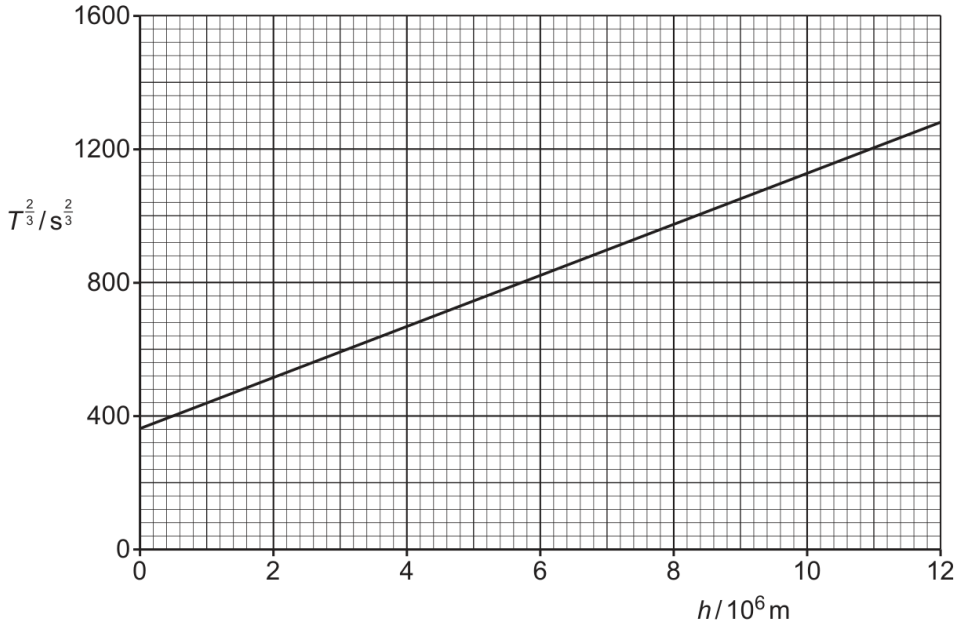


Fig. 1.1

(i) By reference to forces, explain why the orbit of the satellite is circular.

.....

.....

..... [2]

(ii) Use Newton's law of gravitation to show that h and T are related by

$$(h + B)^3 = \frac{GA}{4\pi^2} T^2$$

where G is the gravitational constant and A and B are constants that depend on the properties of the planet.

[3]

(iii) Use the gradient and intercept of the line in Fig. 1.1 to determine values for A and B. Give units with your answers.

A = ... unit .....

B = ... unit .....

[5]

2 **(a)** State Newton's law of gravitation.

.....

.....

..... [2]

(b) A satellite is in a circular orbit around a planet. The radius of the orbit is R and the period of the orbit is T. The planet is a uniform sphere.

Use Newton's law of gravitation to show that R and T are related by

$$4\pi^2R^3 = GMT^2$$

where M is the mass of the planet and G is the gravitational constant.

[2]

(c) The Earth may be considered to be a uniform sphere of mass 5.98×10^{24} kg and radius 6.37×10^6 m.

A geostationary satellite is in orbit around the Earth.

Use the expression in **(b)** to determine the height of the satellite above the Earth's surface.

height = ... m [3]

(d) Another satellite is in a circular orbit around the Earth with the same orbital radius and period as the satellite in **(c)**.

(ii) Despite having the same orbital period, the orbit of this satellite is not geostationary.

Suggest **two** ways in which the orbit of this satellite could be different from the orbit of the satellite in **(c)**.

1 .....

.....

2 .....

.....

[2]

3 **(a)** **(i)** State Newton's law of gravitation.

.....

.....

..... [2]

4 **(a)** State the equation for the gravitational force F between two point masses m_1 and m_2 that are separated by a distance r. State the meaning of any other symbols you use.

[2]

(b) A satellite is in a circular orbit of radius R around a planet of mass M.

Show that the period T of the orbit is given by

$$T^2 = kR^3$$

where k is a constant that depends on the value of M. Explain your reasoning.

[3]

(c) A satellite is in a circular orbit around the Earth with a period of 24 hours.
The mass of the Earth is 6.0×10^{24} kg.

(i) Calculate the radius of the orbit.

radius = ... m [2]

(ii) State the **two** other conditions that must be met for the orbit to be geostationary.

1 .....

.....

2 .....

.....

[2]

5 **(b)** A spherical planet can be considered as a point mass at its centre.

(ii) A satellite is in a circular orbit around the planet.

Explain, with reference to your answer in **(b)(i)**, why the path of the satellite is circular.

.....

.....

..... [2]

6 The Earth may be assumed to be an isolated uniform sphere with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 1200 kg is in a circular orbit about the Earth in the Earth's gravitational field. The period of the orbit is 94 minutes.

(b) Calculate the radius of the orbit of the satellite.

radius = ... m [3]

(c) Rockets on the satellite are fired so that the satellite enters a different circular orbit that has a period of 150 minutes. The change in the mass of the satellite may be assumed to be negligible.

(i) Show that the radius of the new orbit is 9.4×10^6 m.

[2]

7 **(a)** State what is meant by a *gravitational force*.

.....

..... [1]

(b) A binary star system consists of two stars S_1 and S_2, each in a circular orbit.

The orbit of each star in the system has a period of rotation T.

Observations of the binary star from Earth are represented in Fig. 1.1.

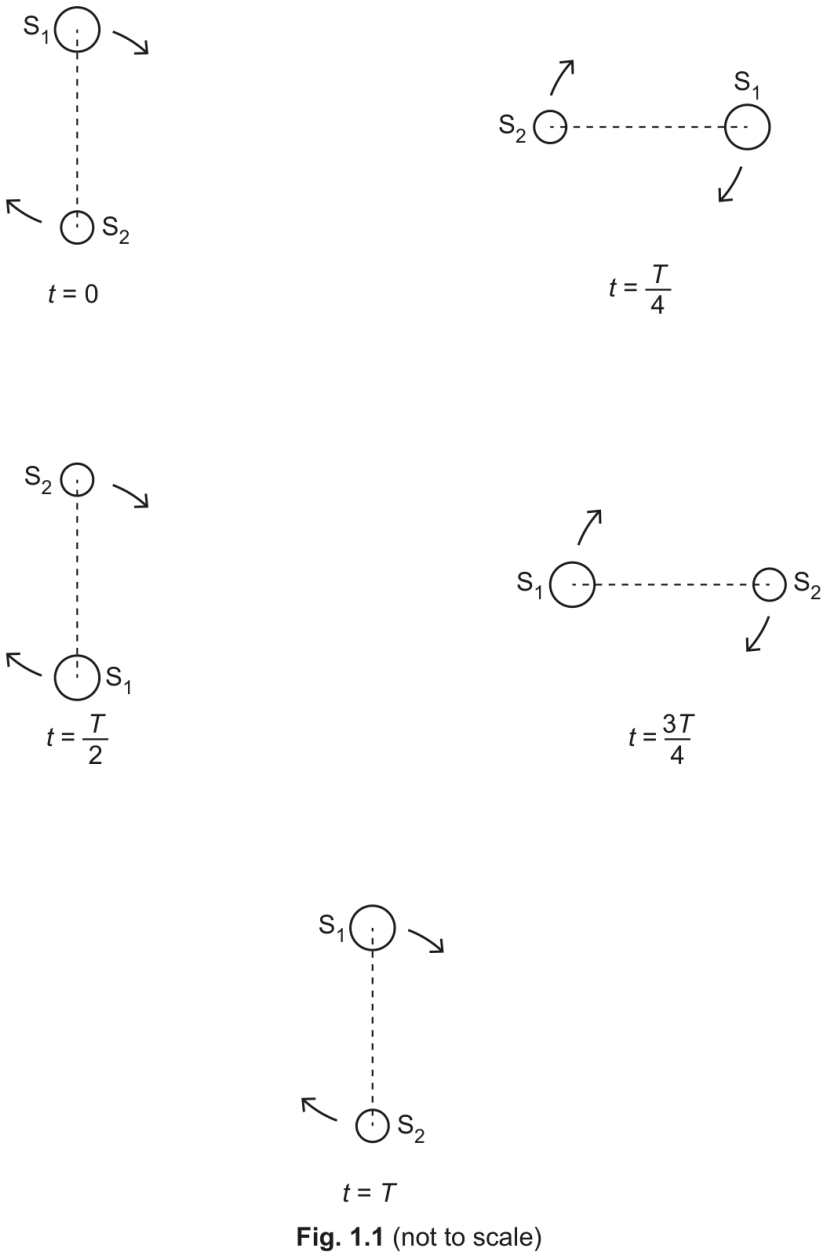


Fig. 1.1 (not to scale)

Observed from Earth, the angular separation of the centres of S_1 and S_2 is 1.2×10^{-5} rad. The distance of the binary star system from Earth is 1.5×10^{17} m.

Show that the separation d of the centres of S_1 and S_2 is 1.8×10^{12} m.

(c) The stars S_1 and S_2 rotate with the same angular velocity ω about a point P, as illustrated in Fig. 1.2.

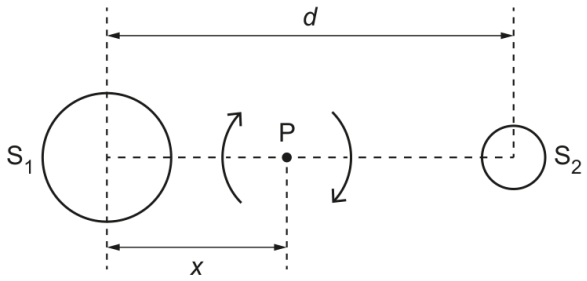


Fig. 1.2 (not to scale)

Point P is at a distance x from the centre of star S_1.
The period of rotation of the stars is 44.2 years.

(i) Calculate the angular velocity ω.

(iii) The mass M_1 of star S_1 is given by the expression

$$GM_1 = d^2(d - x)\omega^2$$

where G is the gravitational constant.

The ratio in **(ii)** is found to be 1.5.

Use data from **(b)** and your answer in **(c)(i)** to determine the mass M_1.

M_1 = ... kg [3]

8 **(b)** The Earth may be considered to be a uniform sphere of radius 6.4×10^6 m with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 2.4×10^3 kg is launched from the Equator. It is placed in an equatorial orbit at a height of 5.6×10^6 m above the Earth's surface.

(ii) Determine the speed of the satellite when in orbit.

speed = ... $m\,s^{-1}$ [3]

(c) Before the satellite in **(b)** is launched, its speed at the Equator due to the Earth's rotation is $470\,m\,s^{-1}$.

Suggest why the energy required to launch the satellite depends on whether the satellite, in its orbit, is travelling from west to east or from east to west.

.....

..... [1]

9 **(a)** Two point masses are isolated in space and are separated by a distance x.

State an expression relating the gravitational force F between the two masses to the magnitudes M and m of the masses. State the name of any other symbol used.

.....

.....

..... [1]

(b) A spacecraft is to be put into a circular orbit about a spherical planet.

The planet may be considered to be isolated in space. The mass of the planet, assumed to be concentrated at its centre, is 7.5×10^{23} kg. The radius of the planet is 3.4×10^{6} m.

(i) The spacecraft is to orbit the planet at a height of 2.4×10^{5} m above the surface of the planet. At this altitude, there is no atmosphere.

Show that the speed of the spacecraft in its orbit is $3.7 \times 10^{3}\,\text{m s}^{-1}$.

[2]

10 (a) Two point masses are separated by a distance x in a vacuum.
State an expression for the force F between the two masses M and m.
State the name of any other symbol used.

.....

.....

.....[1]

(b) A small sphere S is attached to one end of a rod, as shown in Fig. 1.1.

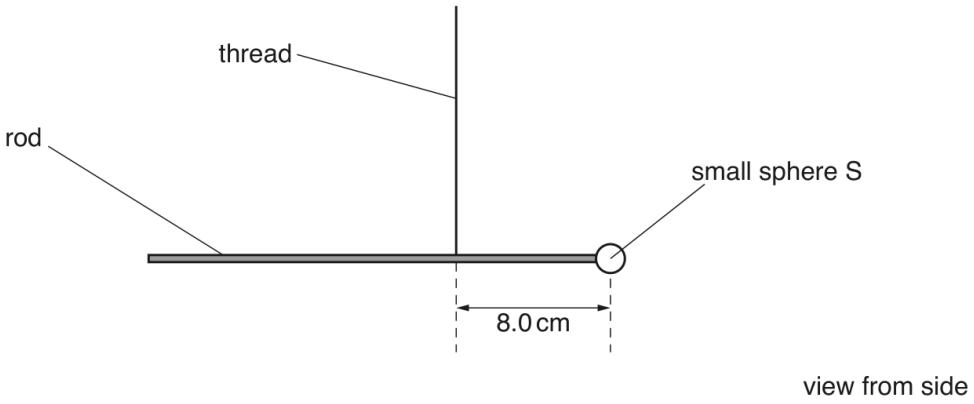


Fig. 1.1 (not to scale)

The rod hangs from a vertical thread and is horizontal.
The distance from the centre of sphere S to the thread is 8.0 cm.

A large sphere L is placed near to sphere S, as shown in Fig. 1.2.

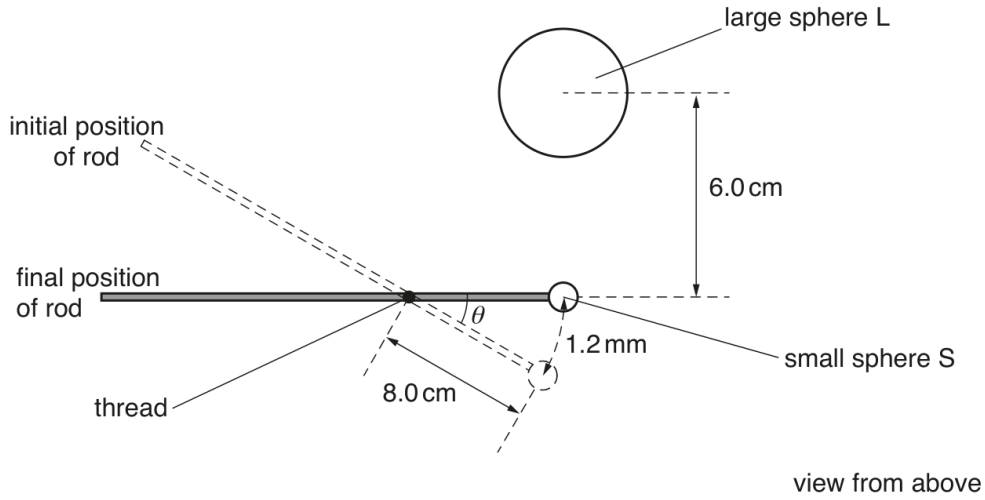


Fig. 1.2 (not to scale)

There is a force of attraction between spheres S and L, causing sphere S to move through a distance of 1.2 mm.
The line joining the centres of S and L is normal to the rod.

(ii) The rotation of the rod causes the thread to twist.
The torque T (in N m) required to twist the thread through an angle β (in rad) is given by

$$T = 9.3 \times 10^{-10} \times \beta.$$

Calculate the torque in the thread when sphere L is positioned as shown in Fig. 1.2.

torque = ... N m [1]

(c) The distance between the centres of spheres S and L is 6.0 cm.
The mass of sphere S is 7.5 g and the mass of sphere L is 1.3 kg.

(i) By equating the torque in **(b)(ii)** to the moment about the thread produced by gravitational attraction between the spheres, calculate a value for the gravitational constant.

gravitational constant = ... $N m^2 kg^{-2}$ [3]

(ii) Suggest why the total force between the spheres may not be equal to the force calculated using Newton's law of gravitation.

.....

.....[1]

11 **(a)** State Newton's law of gravitation.

.....

.....

..... [2]

(b) A geostationary satellite orbits the Earth. The orbit of the satellite is circular and the period of the orbit is 24 hours.

(i) State **two** other features of this orbit.

1. .....

.....

2. .....

.....

[2]

(ii) The radius of the orbit of the satellite is 4.23×10^4 km.

Determine a value for the mass of the Earth. Explain your working.

mass = ... kg [4]

12 (a) State Newton's law of gravitation.

.....

.....

..... [2]

(b) The astronomer Johannes Kepler showed that the period T of rotation of a planet about the Sun is related to its mean distance R from the centre of the Sun by the expression

$$\frac{R^3}{T^2} = k$$

where k is a constant.

Use Newton's law to show that, for planets in circular orbits about the Sun of mass M, the constant k is given by

$$k = \frac{GM}{4\pi^2}$$

where G is the gravitational constant. Explain your working.

[4]

(c) A satellite is in a circular orbit about Mars.
The radius of the orbit of the satellite is 4.38×10^6 m. The orbital period is 2.44 hours.

Use the expressions in **(b)** to calculate a value for the mass of Mars.

mass = ... kg [2]

13 **(a)** State Newton's law of gravitation.

.....

.....

.....

.....[2]

(b) A distant star is orbited by several planets. Each planet has a circular orbit with a different radius.

(ii) The radius of the orbit of a planet is R and the orbital period is T.

Data for some of the planets are given in Fig. 1.1.

planet	R/m	T^2/s^2
c	9.6×10^{10}	2.5×10^{11}
e	4.0×10^{11}	1.8×10^{13}
g	2.1×10^{12}	2.6×10^{15}

Fig. 1.1

The relationship between R and T is given by the expression

$$R^3 = kT^2.$$

1. Show that the constant k is given by the expression

$$k = \frac{GM}{4\pi^2}$$

where G is the gravitational constant and M is the mass of the star.

[3]

2. Use data from Fig. 1.1 for the three planets and the expression for k to calculate the mass M of the star.

M = ... kg [3]